Noise Effects in Various Quantitative Susceptibility Mapping Methods

Zhou Saiya
2014.11.3

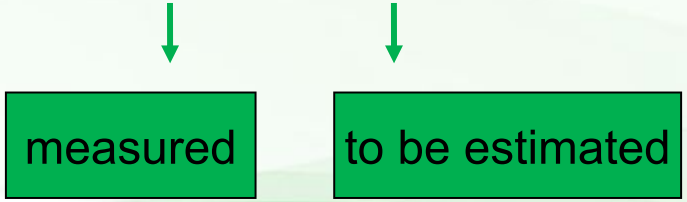
Shuai Wang, Tian Liu, Weiwei Chen, Pascal Spincemaille, Cynthia Wisnieff, A. John Tsiouris, Wenzhen Zhu, Chu Pan, Lingyun Zhao, and Yi Wang ,2013(cited：4)

Various regularization methods have been proposed for QSM. In this paper, various QSM methods were systematically compared for noise analysis.

Quantitative Susceptibility Mapping (QSM)

Estimation of the susceptibility map χ from the unwrapped phase ϕ

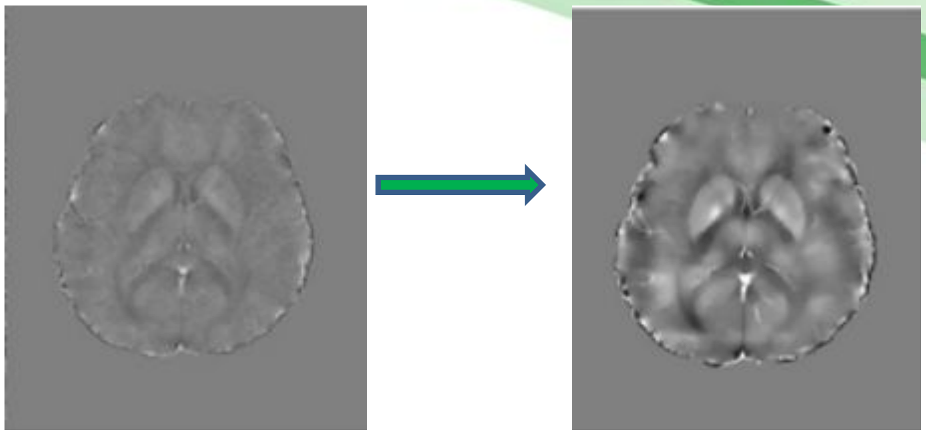
$$\delta = d * \chi \qquad (1)$$



measured

to be estimated

d: susceptibility dipole kernel



$$\delta = \frac{\varphi}{\gamma \cdot TE \cdot B_0}$$

According to Eq.(1),we can solve for x by convolving with the inverse of D

$$\delta = d * \chi$$

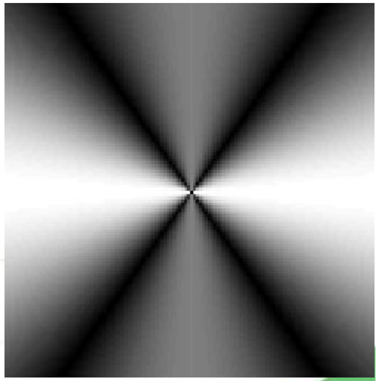
⇓

$$\Delta(k) = D(k) \cdot X(k)$$

⇓

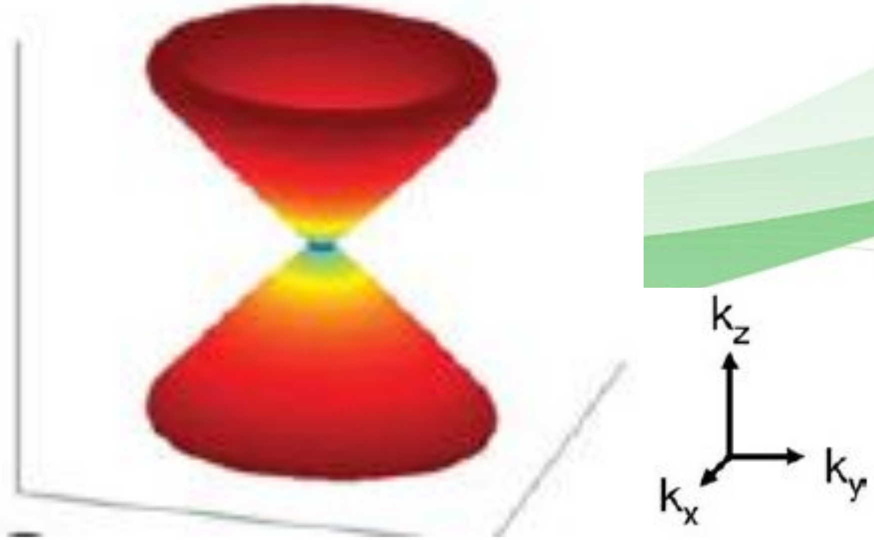
$$X(k) = D(k)^{-1} \cdot \Delta(k)$$

The inversion is made difficult by zeros on a conical surface in susceptibility kernel



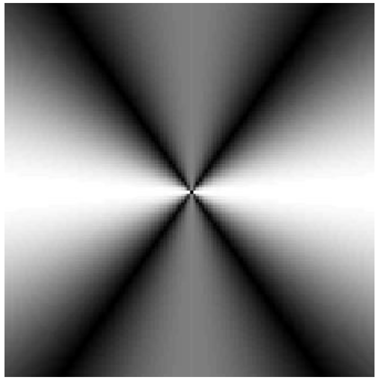
$|D|$

$$D(k) = \frac{1}{3} - \frac{k_z^2}{k^2}$$



TKD: truncated k-space division

$$X = \begin{cases} \dfrac{\Delta}{D} & ,|D| \geq \tau \\ sign(D)\dfrac{\Delta}{\tau} & ,|D| < \tau \end{cases}$$



$|D|$

τ is a small predetermined threshold

Magnetic Susceptibility Mapping of Brain Tissue In Vivo Using MRI Phase Data,MRM,2009(cited:151)

WKD: weighted k-space derivative

To calculate the x (k) values on the conical surfaces, we utilize the first-order derivatives of Eq. (2). The rationale is that although D(k) =0 on the conical surfaces, its derivative is not.

$$\Delta(k) = D(k)\cdot X(k)$$

$$\Delta'(k) + [2(k_x^2 + k_y^2)k_z / k^4]\cdot X(k) - D(k)\cdot X'(k) = 0$$

Quantitative susceptibility mapping of human brain reflects spatial variation in tissue composition,Neuroimage,2011(cited:96)

WKD

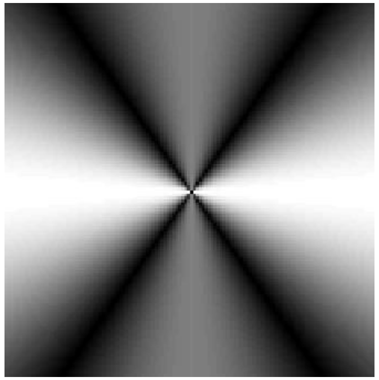
$$\Delta'(k)+[2(k_x^2+k_y^2)k_z/k^4]\cdot X(k)-D(k)\cdot X'(k)=0$$

$$\Bigg\downarrow \quad \begin{aligned} &D_2(k)=(k_x^2+k_y^2)k_z/\pi k^4 \\ &\Delta'(k)=FT[-i2\pi r_z\delta] \\ &X'(k)=FT[-i2\pi r_z\chi] \end{aligned}$$

$$D_2(k)\cdot X(k)+\underline{D(k)\cdot FT[i\cdot r_z\chi]}=FT[i\cdot r_z\delta]$$

$$D_2(k)\cdot X(k)=FT[i\cdot r_z\delta]$$

WKD



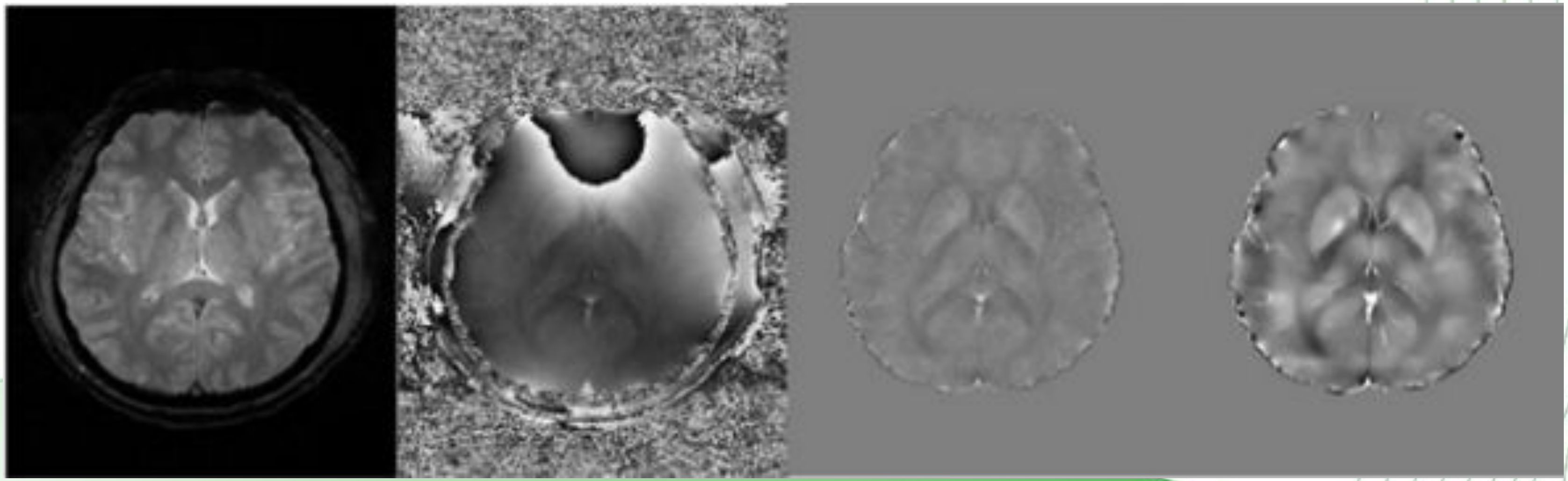
$|D|$

$$X(k)=\begin{cases} D(k)^{-1}\cdot\Delta(k) & ,|D|\geq\varepsilon \\ M_{D3}\cdot D_2(k)^{-1}\cdot FT[i\cdot r_z\delta], & |D|<\varepsilon \end{cases}$$

M_{D3} is smooth weighting function ,which is to emphasize the derivative relationship near the conical surfaces.

MEDI: Morphology enabled dipole

The MEDI method makes use of the observation that the locations of the interfaces (or edges) in the susceptibility distribution are nearly the same as those in magnitude images obtained in the same acquisition, we consider their discordance to be sparse.



Morphology Enabled Dipole Inversion (MEDI) from a Single-Angle Acquisition: Comparison with COSMOS in Human Brain Imaging, MRM,2011(cited:60)

MEDI: Morphology enabled dipole

To promote this sparsity, penalize susceptibility at those voxels that are not part of an interface in the magnitude image.

$$\min_{\chi} \|\mathrm{M}G\chi\|_1 \; s.t. \|W(\delta - d * \chi)\|_2 = \varepsilon$$

M is a binary mask $$M = \begin{cases} 0 & |Gm| \geq \mu \\ 1 & |Gm| < \mu \end{cases}$$

$$\chi^* = \mathrm{argmin}_{\chi} \|MG\chi\|_1 + \lambda \|W(\delta - d * \chi)\|_2^2$$

TV&WA method

$$\chi = \arg\min_{\chi} \left\|\delta - d * \chi\right\|_2^2 + \underbrace{\alpha \left\|\Phi\chi\right\|_1 + \beta TV(\chi)}_{\text{cost function}}$$

Whole Brain Susceptibility Mapping Using Compressed Sensing, MRM, 2012, (cited:54)

Work in this paper

D has values that are infinitesimal in the vicinity of the cone surfaces, which will lead to large noise amplification upon inversion.

$$\delta_b = d * \chi + n$$

$$\Big\Downarrow$$

$$\Delta_b = DX + N$$

$$\mathbf{D} = \left(\frac{1}{3} - \frac{\boldsymbol{k}_z^2}{\boldsymbol{k}^2} \right)$$

If the noise n had a uniform variance in space and could be modeled by independent and identically distributed Gaussian random variables.

$$E = \|z\|_2^2, \text{with } z = d * \chi - \delta_b$$

This estimated Gaussian noise is not spatially uniform

$$E = \|wz\|_2^2, \text{with } z_n = \exp[-i(d * \chi)] - \exp(-i\delta_b)$$

Signal-to-noise in phase angle reconstruction: Dynamic range extension using phase reference offsets, MRM, 1990(cited:131)

Four QSM algorithms

TKD $X = H\Delta_b / D + (1 - H) sign(D)\Delta_b / \tau$

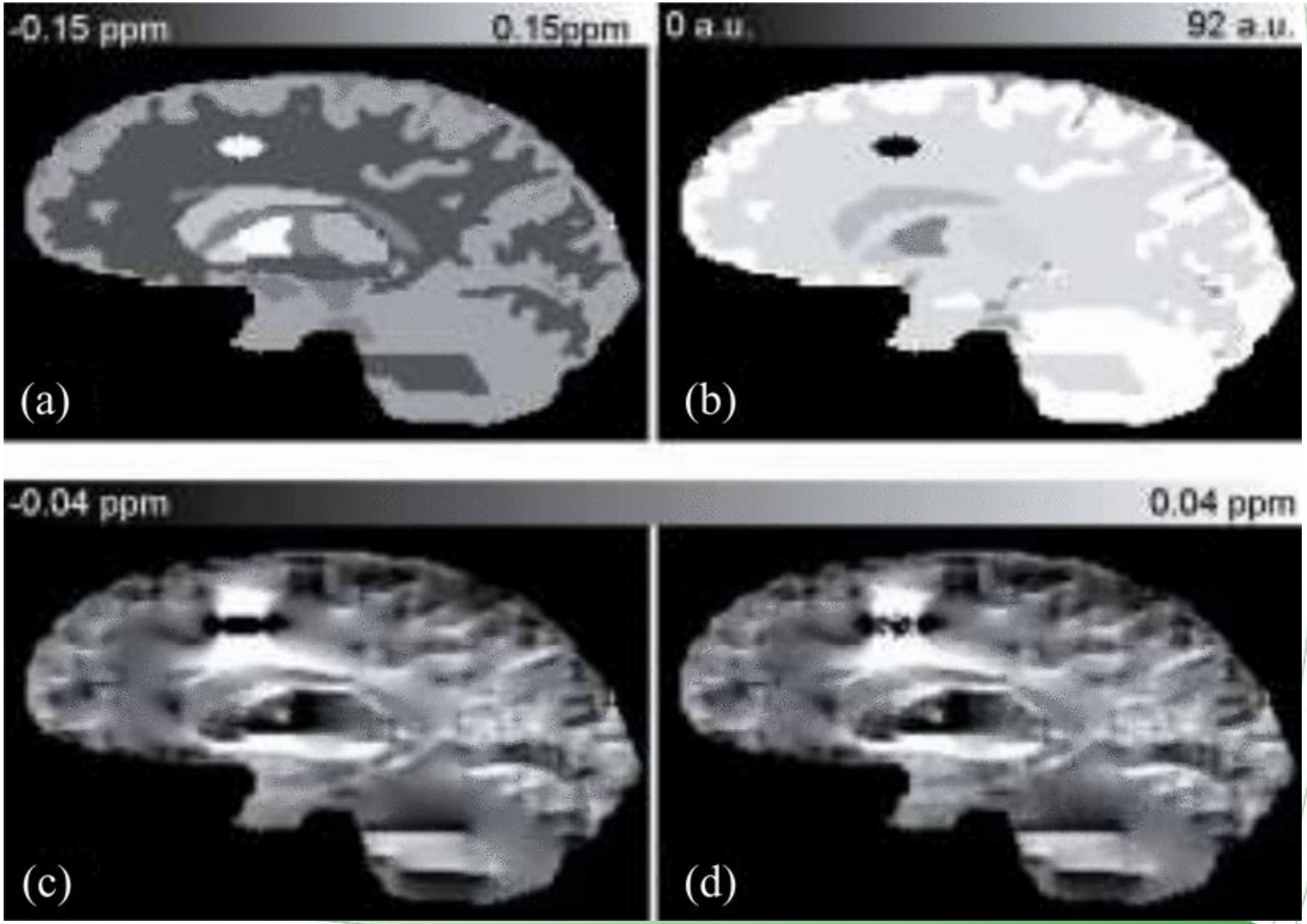
WKD $X = H \cdot \arg\min \|Z\|_2^2 + (1 - H)\arg\min \|M_{D3}Z'\|_2^2$

TVWA $\chi = \arg\min(\|z\|_2^2 + \alpha\|\Phi\chi\|_1 + \beta TV(\chi))$

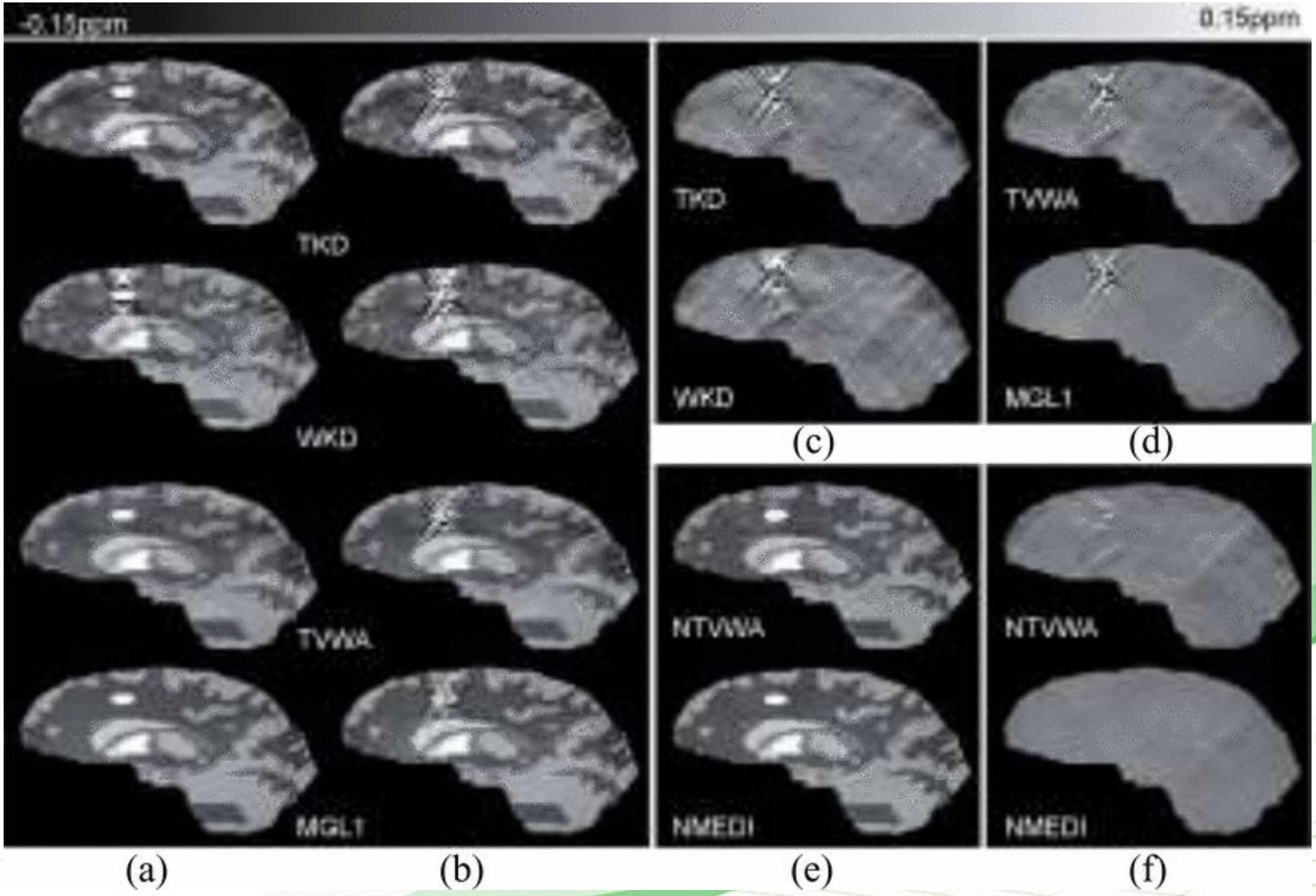
NTVWA $\chi = \arg\min(\|wz_n\|_2^2 + \alpha\|\Phi\chi\|_1 + \beta TV(\chi))$

MGL1 $\chi = \arg\min(\|z\|_2^2 + \alpha\|MG\chi\|_1)$

NMEDI $\chi = \arg\min(\|wz_n\|_2^2 + \alpha\|MG\chi\|_1)$

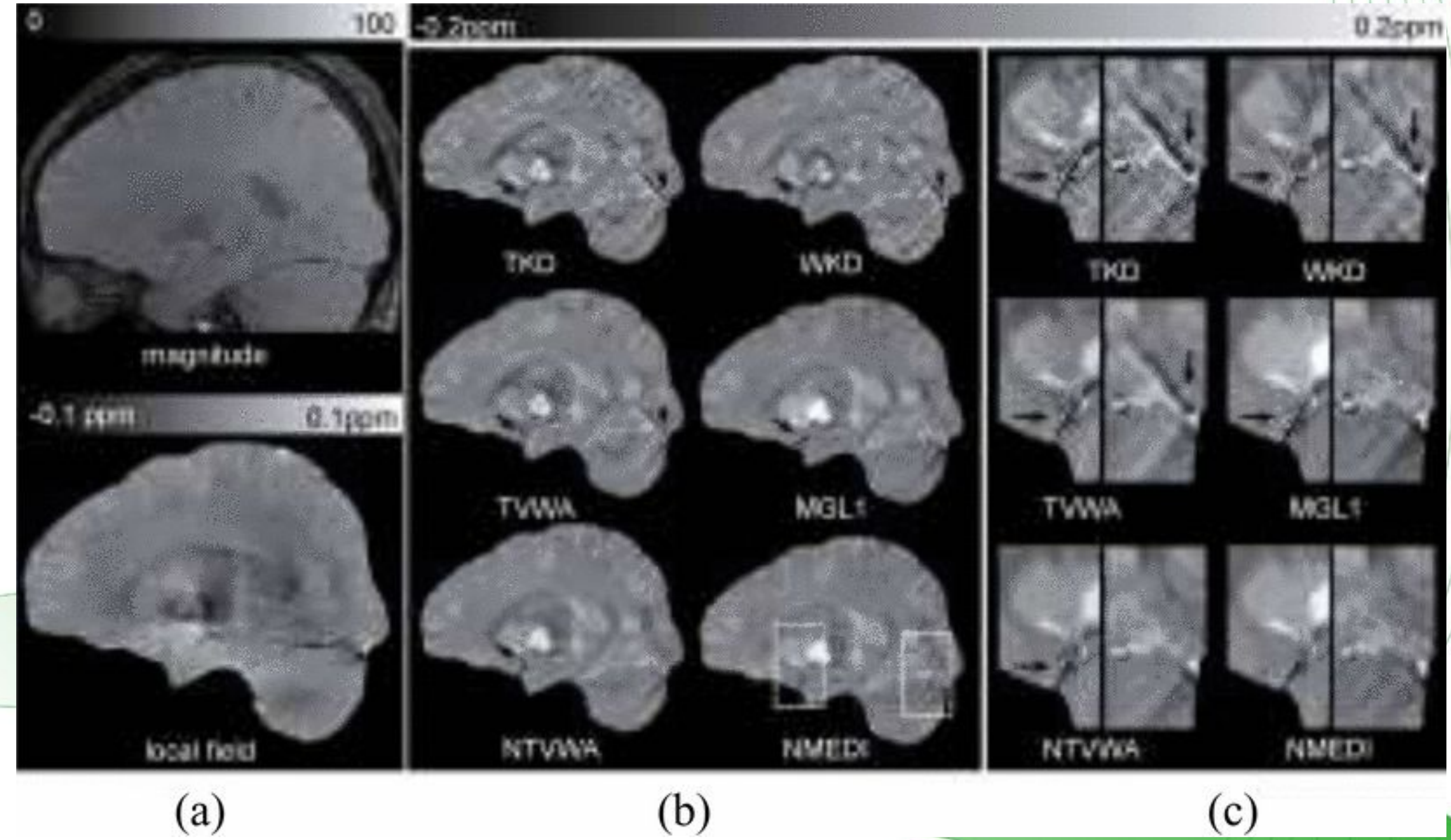


Human brain simulation. (a) True susceptibility map and (b) Magnitude image are shown in the top row. (c)Noiseless field map and (d) Noisy field map are shown in the bottom row.

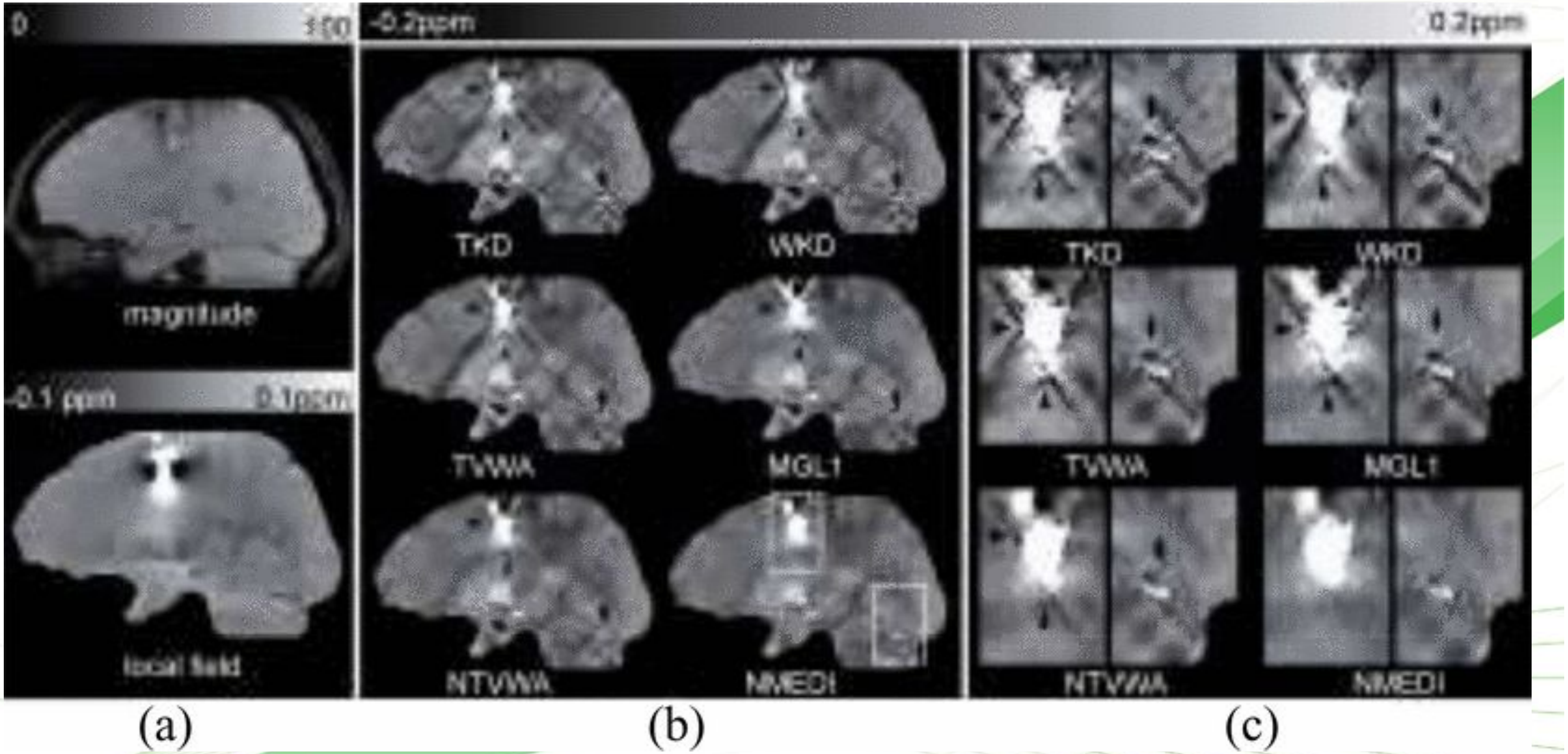


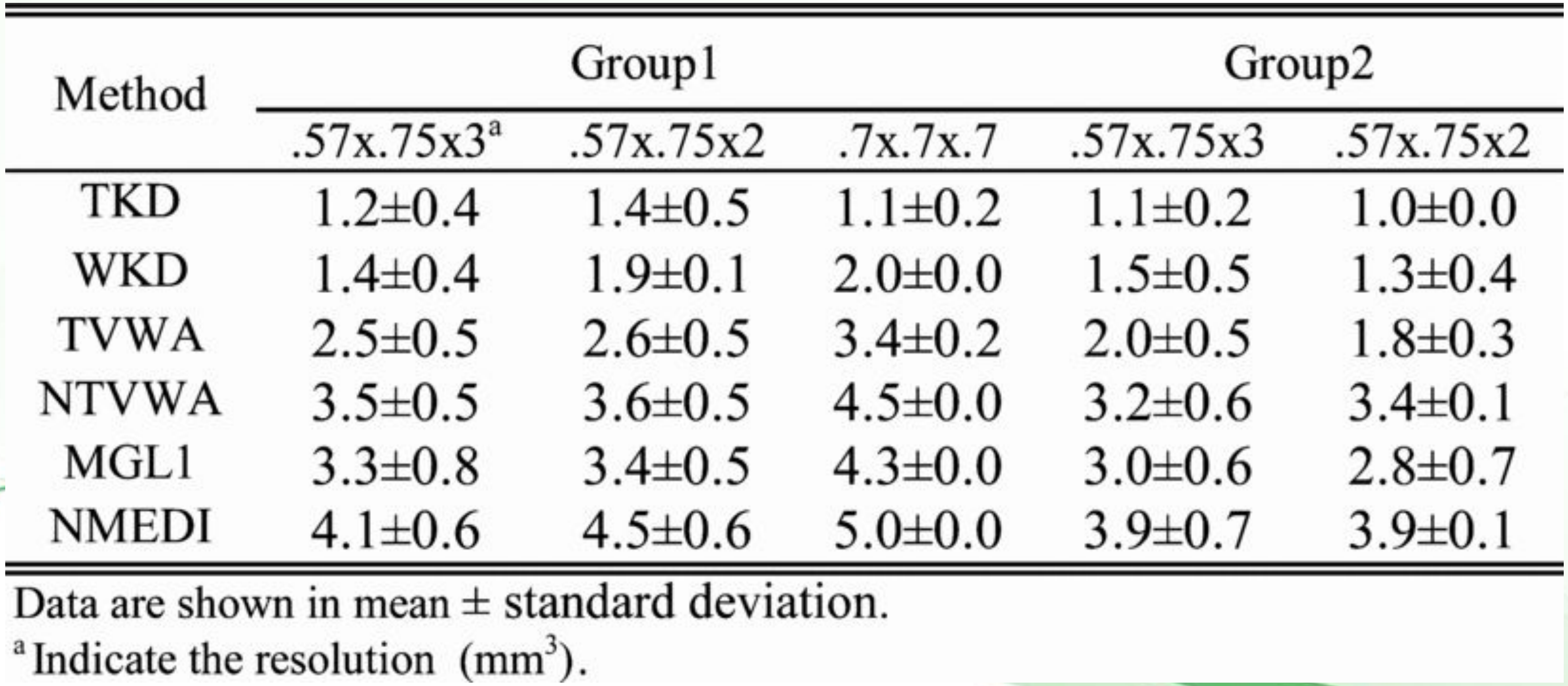
Noise and noise weighting effects of the simulated maps in QSM methods.

Noise weighting effect in QSM on a healthy subject



Noise weighting effects in QSM on a hemorrhage patient





Method	Group1			Group2	
	.57x.75x3[a]	.57x.75x2	.7x.7x.7	.57x.75x3	.57x.75x2
TKD	1.2±0.4	1.4±0.5	1.1±0.2	1.1±0.2	1.0±0.0
WKD	1.4±0.4	1.9±0.1	2.0±0.0	1.5±0.5	1.3±0.4
TVWA	2.5±0.5	2.6±0.5	3.4±0.2	2.0±0.5	1.8±0.3
NTVWA	3.5±0.5	3.6±0.5	4.5±0.0	3.2±0.6	3.4±0.1
MGL1	3.3±0.8	3.4±0.5	4.3±0.0	3.0±0.6	2.8±0.7
NMEDI	4.1±0.6	4.5±0.6	5.0±0.0	3.9±0.7	3.9±0.1

Data are shown in mean ± standard deviation.

[a] Indicate the resolution (mm^3).

Thank you!!!