文献报告

2014.11.10

An Iterative Spherical Mean Value Method for Background Field Removal in MRI

MRM 2014 cited：3

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- Introduction
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- Experiments and Results
- Conclusion

QSM reconstruction

1.the input data of QSM:

Gradient echo (GRE) phase, has recently been shown to have substantially improved signal-to-noise characteristics compared to GRE magnitude images at high and ultrahigh field strength

2.processing steps:

(1) phase unwrapping

(2) background field removal

（3） Dipole inversion

Frequency map estimation → Spatial frequency unwrapping → Background filed removal → Dipole inversion

Introduction

Three commonly used methods in removing background field in QSM:

1.HPF

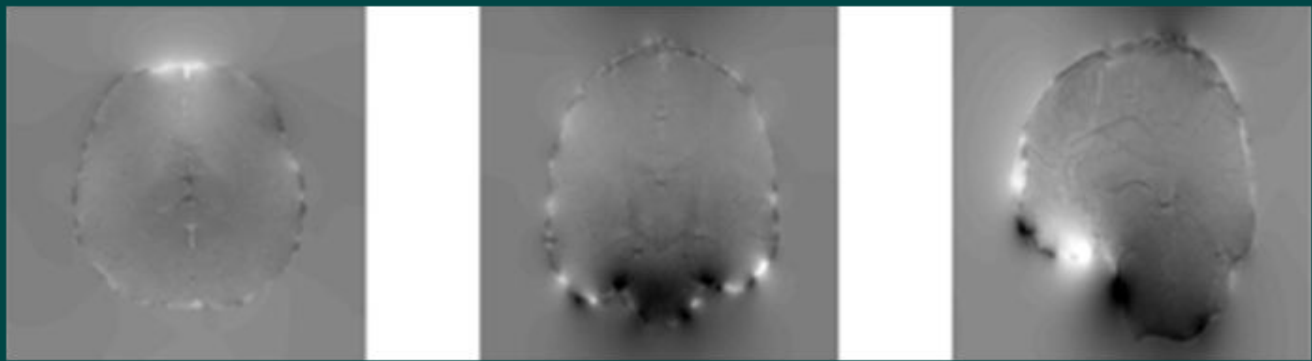
2.PDF

3.SHARP

background field removal (sharp)

$f = f_B + f_L$

- Since f_B results from sources that are located outside the VOI, f_B is harmonic throughout the entire VOI.



Quantitative imaging of intrinsic magnetic tissue properties using MRI signal phase: an approach to in vivo brain iron metabolism?
Schweser F, Deistung A, Lehr BW, Reichenbach JR.
Neuroimage. 2011 Feb 14;54(4):2789-807.(cited 137)

SHARP法

$$B^{\Delta}(\mathrm{r}) = B_{\mathrm{int}}(\mathrm{r}) + B_{ext}(\mathrm{r})$$

依据： B_{ext} (r)是由VOI外的源所引起的场，所以B_{ext} (r) 在VOI内是调和函数因此在VOI内有：

$$\nabla^2 B_{\mathrm{ext}} = 0$$

↓

$$\nabla^2 B^{\Delta} = \nabla^2 (B_{\mathrm{int}} + B_{ext}) = \nabla^2 B_{\mathrm{int}}$$

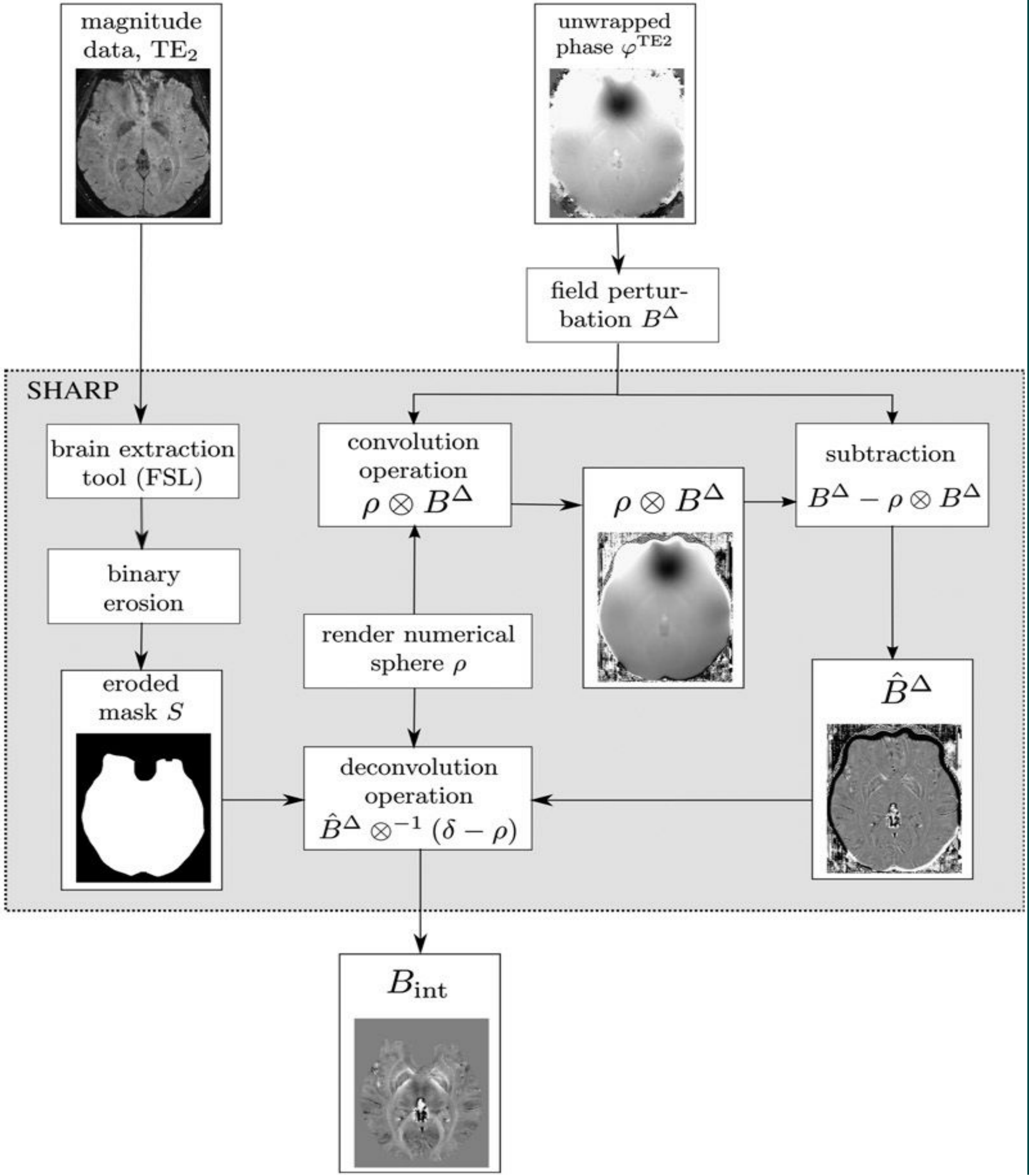
利用调和函数的均值理论可以把一个调和函数表示为它与一个径向函数ρ(r)的卷积：

$$u(\vec{r}) = (\rho \otimes u)(\vec{r})$$

$$\begin{aligned}\hat{B}^{\Delta} &= B^{\Delta} - \rho \otimes B^{\Delta} \\ &= B_{\text{ext}} + B_{\text{int}} - \rho \otimes B_{\text{int}} - \rho \otimes B_{\text{ext}} \\ &= B_{\text{int}} - \rho \otimes B_{\text{int}}.\end{aligned}$$

$$\hat{B}^{\Delta} = B_{\text{int}} - \rho \otimes B_{\text{int}} = (\delta - \rho) \otimes B_{\text{int}},$$

$$B_{\text{int}} = (\delta - \rho) \otimes^{-1} \hat{B}^{\Delta}.$$



magnitude data, TE_2
unwrapped phase φ^{TE2}
field perturbation B^{Δ}
SHARP
brain extraction tool (FSL)
convolution operation $\rho \otimes B^{\Delta}$
$\rho \otimes B^{\Delta}$
subtraction $B^{\Delta} - \rho \otimes B^{\Delta}$
binary erosion
render numerical sphere ρ
eroded mask S
$\hat{B}^{\Delta}$
deconvolution operation $\hat{B}^{\Delta} \otimes^{-1} (\delta - \rho)$
B_{int}

SHARP

The SHARP method is a three-step procedure of convolution with a radial kernel, masking, and deconvolution.

convolution $$(\delta-\rho)\otimes \mathrm{f} = (\delta-\rho)\otimes \mathrm{f}_{\mathrm{L}}$$

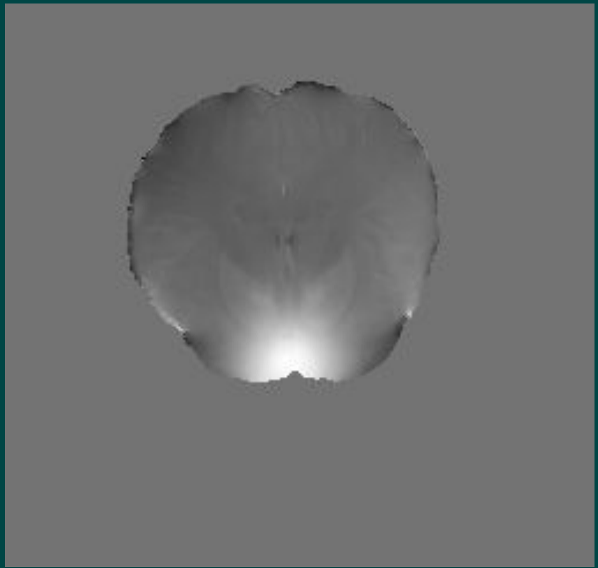
⇓

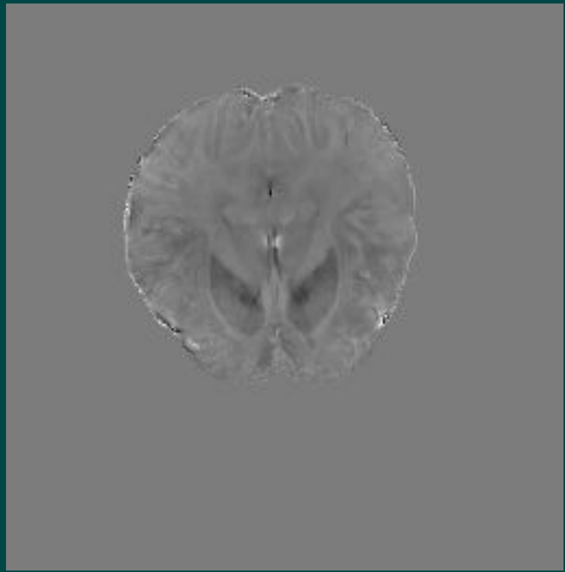
masking $$M(\delta-\rho)\otimes \mathrm{f} = M(\delta-\rho)\otimes \mathrm{f}_{\mathrm{L}}$$

⇓

deconvolution
use TSVD $$F^{-1}CF\mathrm{f}_{\mathrm{L}} = MF^{-1}CF\mathrm{f}$$

sharp





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Theory and Methods

The SHARP method relies on the fact that a harmonic field satisfies the following property: For any given sphere at any given location within the ROI, the mean value of a harmonic function over that sphere is equal to the function value at the center of that sphere

The local field obtained with the SHARP method depends on the radius used in the SMV operation and on the threshold needed for the subsequent deconvolution. Additionally, because the SMV cannot be computed for voxels that are less than a radius away from the ROI border, the local field is only available on a modified ROI that is smaller than the original ROI

The Sophisticated Harmonic Artifact Reduction for Phase

The SMV Operation

$$(SMV_r f)(\mathbf{x_0}) = \frac{1}{V}\int_{B(\mathbf{x_0},r)} f(x)\,dV$$

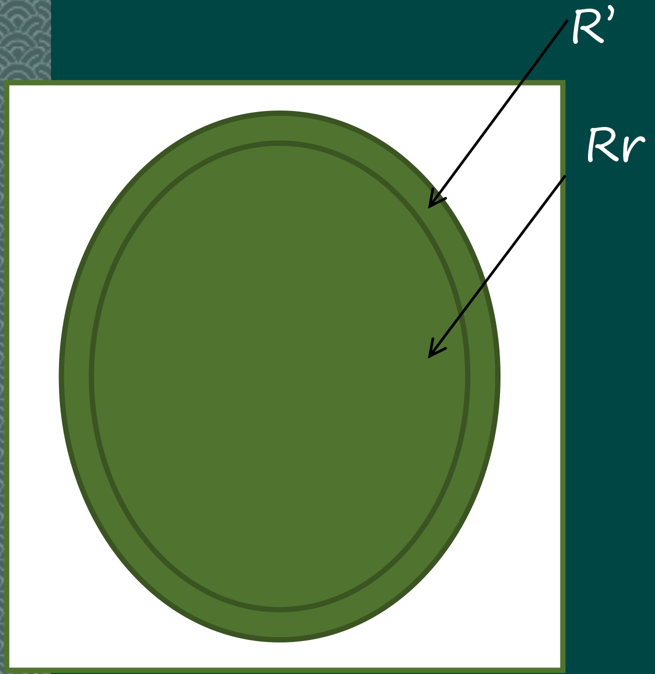
$$f(\mathbf{x_0}) = (SMV_r f)(\mathbf{x_0})$$

Note that for points near the boundary of R, the SMVr operation cannot be faithfully calculated, since it would require data outside R, which we assume is not available.

High-Precision Mapping of the Magnetic Field Utilizing the Harmonic Function Mean Value Property

Journal of Magnetic Resonance 148, 442–448 (2001) 被引用24次

The Sophisticated Harmonic Artifact Reduction for Phase



ROI:

1) Rr : the region where the SMVr operation does not require data outside R
2) R': the region whose voxels are less than a distance r away from the boundary of R.

The Sophisticated Harmonic Artifact Reduction for Phase

The SHARP Method：

$$F_{nh} = (1 - SMV_r)^{-1} M_r (1 - SMV_r) F_0$$

the convolution of the total field F_0 with the （1- SMVr） kernel removes the harmonic background field , the point-to-point multiplication with Mr sets the value of all voxels outside of Rr to zero , and the deconvolution（1-SMVr）$^{-1}$ is computed in Fourier space using a point-to-point division with the Fourier transform of the （1-SMVr）kernel . This kernel vanishes at k =0 , such that truncation is needed before applying the division to avoid noise amplification.

Iterative Spherical Mean Value Method

For functions f defined on the ROI R , we define the operator Sr as follows:

$$(S_r f)(\mathbf{x}) = \begin{cases} f(\mathbf{x}) & \mathbf{x} \in R' \\ (SMV_r f)(\mathbf{x}) & \mathbf{x} \in R_r \end{cases}$$

make the assumption that the background field F_h is known on the border R'. Define a modified field map F that is equal to F_h on the border R' and equal to F_0 on the inner (cropped) region Rr

$$F_0 = \begin{cases} F_h & x \in R' \\ F_0 & x \in R_r \end{cases}$$

Iterative Spherical Mean Value Method

Define the function $S_r^{\infty} F_0$ as the result of applying the operator Sr an infinite number of times on F .

Then function satisfies:

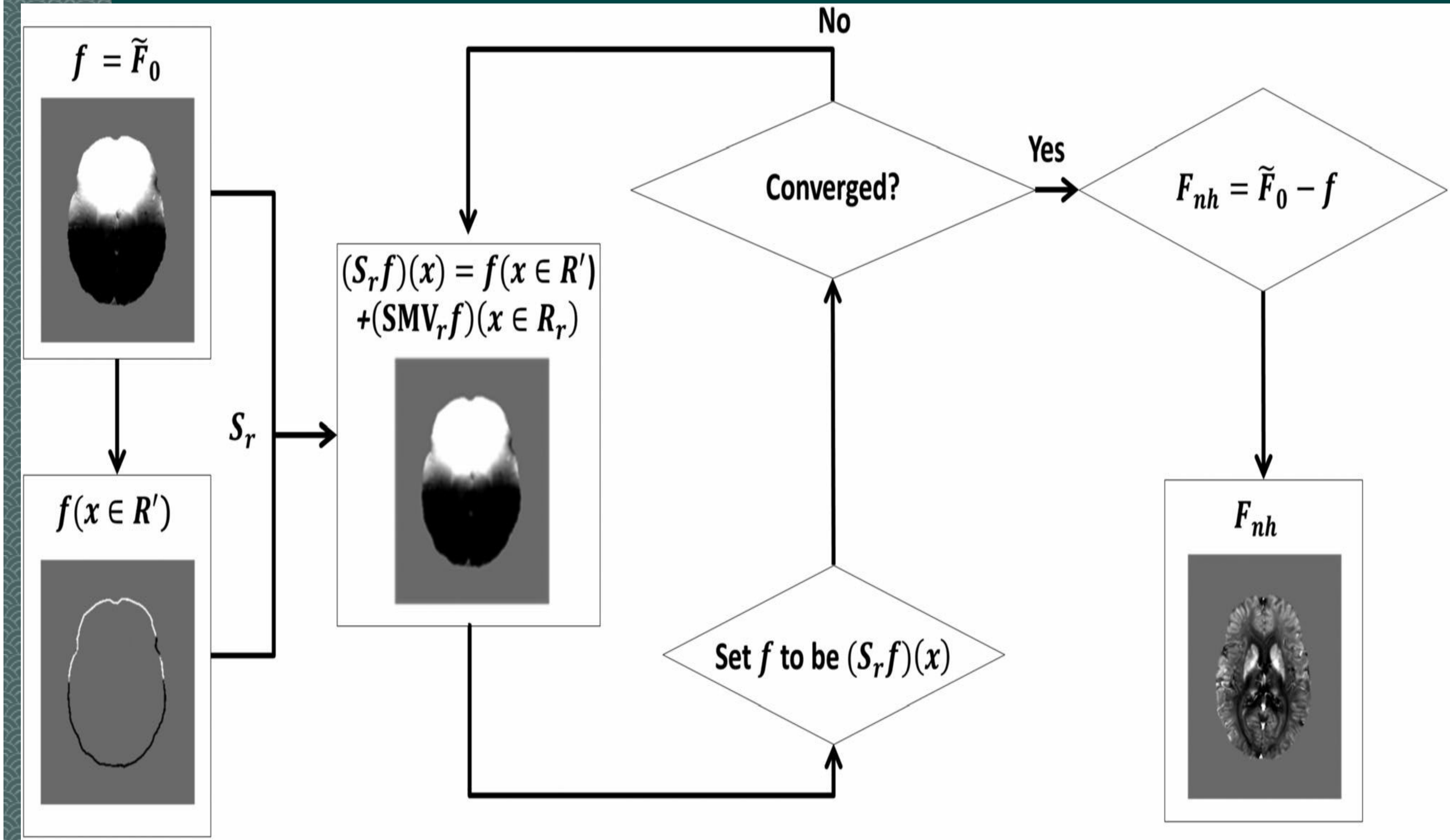
1. it is equal to F_{h} on the border R' by definition
2. For all $\mathbf{x} \in R_r : (\mathrm{SMV_r}(S_r^{\infty} F_0))(\mathbf{x}) = (\mathrm{SMV_r}(\mathrm{SMV}_r^{\infty} \tilde{\mathrm{F}}_0))(\mathbf{x})$

$$=(\mathrm{SMV_r}^{\infty} F_0) = (S_r^{\infty} F_0)(\mathbf{x})$$

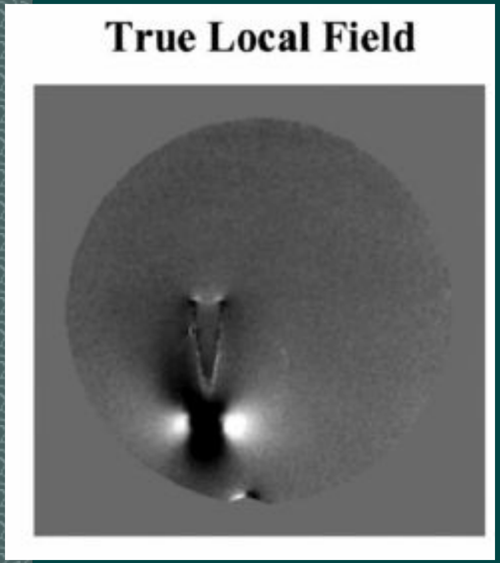
such that $S_r^{\infty} F_0$ satisfies the SMV property for radius r everywhere inside Rr . Therefore, $S_r^{\infty} F_0$ must be a harmonic function itself. Since the solution to the Dirichlet problem of finding a harmonic function inside a given region that satisfies a given boundary condition is unique for a closed ROI， So

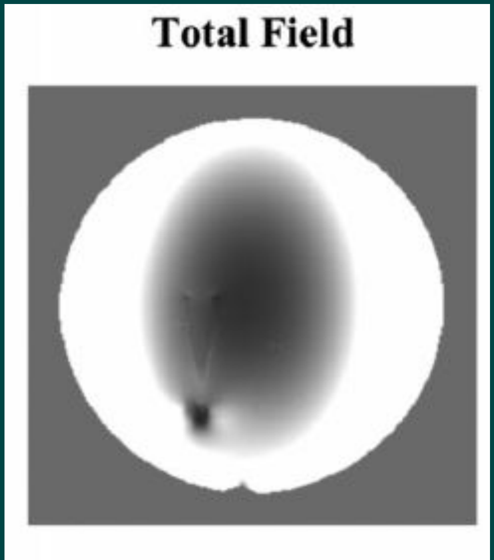
$$F_h = S_r^{\infty} F_0 \qquad\qquad F_{nh} = F_0 - S_r^{\infty} F_0$$

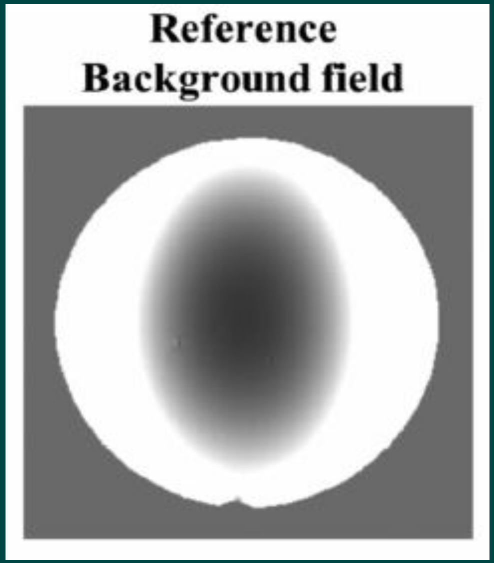
Iterative Spherical Mean Value Method



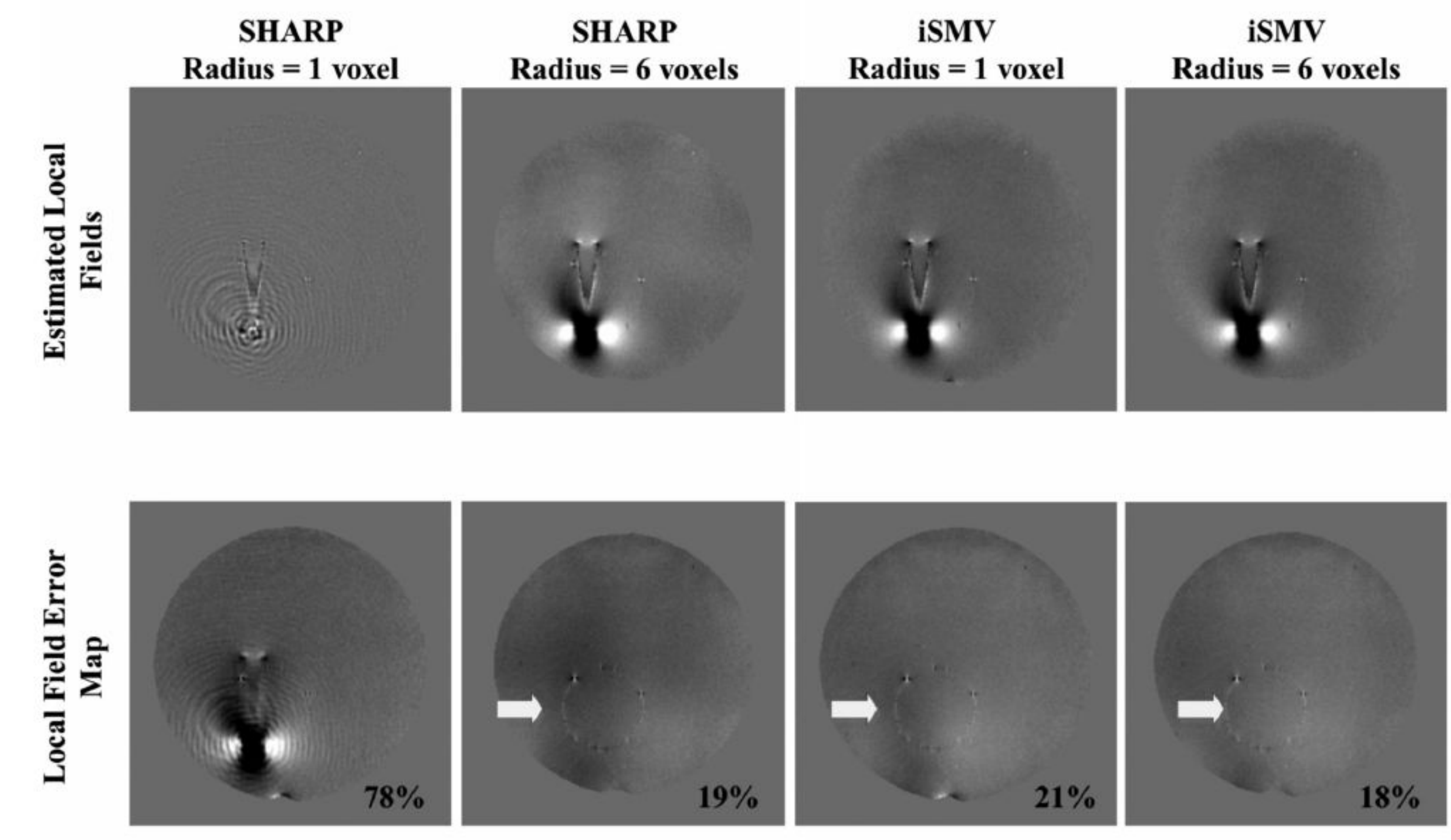
Experiments and Results



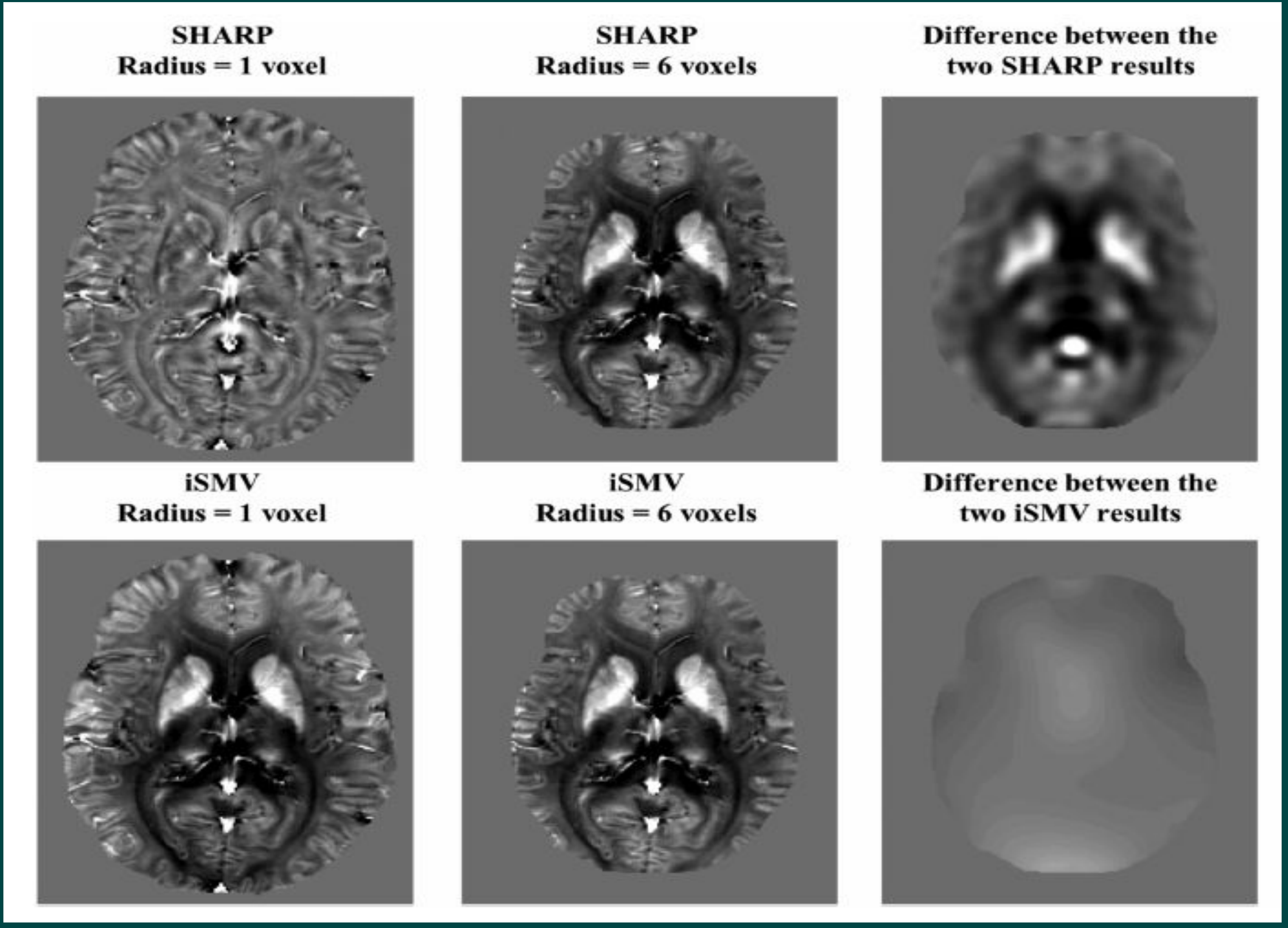




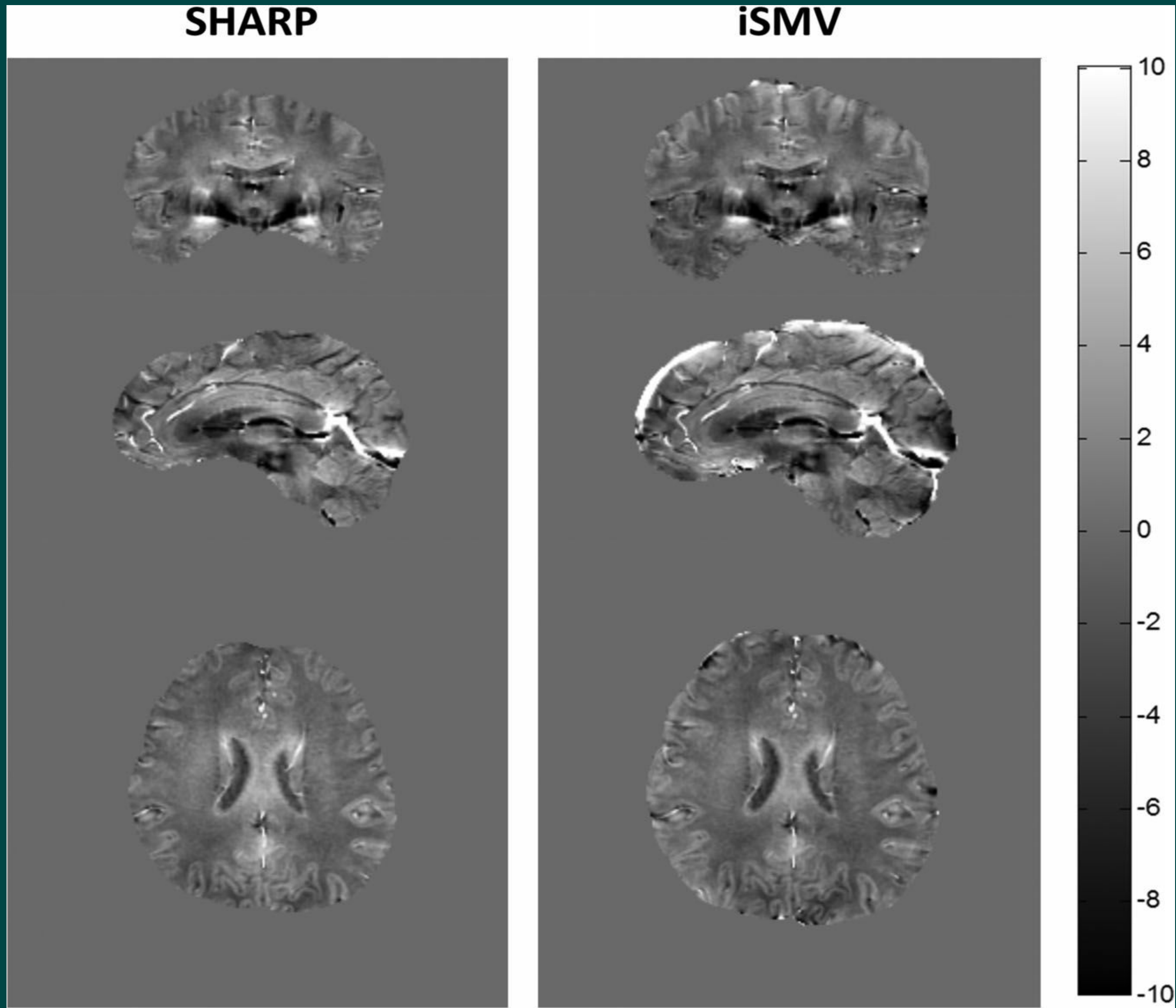
Experiments and Results



Experiments and Results



Experiments and Results



Conclusion

the iterative spherical mean value (iSMV) method is accurate for background field removal in MRI. When compared with SHARP, the dependence of the estimated local field on the radius used for SMV is strongly reduced and allows the estimation of the local field on a larger region of interest.

Thank you！

附录

The mean value property [edit]

If $B(x, r)$ is a ball with center x and radius r which is completely contained in the open set $\Omega \subset \mathbb{R}^n$, then the value $u(x)$ of a harmonic function $u: \Omega \to \mathbb{R}$ at the center of the ball is given by the average value of u on the surface of the ball; this average value is also equal to the average value of u in the interior of the ball. In other words

$$u(x) = \frac{1}{n\omega_n r^{n-1}} \int_{\partial B(x,r)} u \, d\sigma = \frac{1}{\omega_n r^n} \int_{B(x,r)} u \, dy$$

where ω_n is the volume of the unit ball in n dimensions and σ is the n-1 dimensional surface measure .

Conversely, all locally integrable functions satisfying the (volume) mean-value property are both infinitely differentiable and harmonic.

In terms of convolutions, if

$$\chi_r := \frac{1}{|B(0,r)|}\chi_{B(0,r)} = \frac{1}{\omega_n r^n}\chi_{B(0,r)}$$

denotes the characteristic function of the ball with radius r about the origin, normalized so that $\int_{\mathbb{R}^n} \chi_r \, dx = 1$, the function u is harmonic on Ω if and only if

$$u(x) = u * \chi_r(x)$$

as soon as $B(x, r) \subset \Omega$.